

**New Jersey**



***MARCH 2005  
HIGH SCHOOL  
PROFICIENCY ASSESSMENT***

**STUDENT PREPARATION  
BOOKLET**



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## **STUDENT PREPARATION BOOKLET**

### **WHAT IS THE HIGH SCHOOL PROFICIENCY ASSESSMENT?**

In March 2005, all students who entered the eleventh grade for the first time ON or AFTER September 1, 2001, will take the High School Proficiency Assessment (HSPA). The HSPA will measure your knowledge and skills in the Core Curriculum Content Standards, which are designed to make sure that you have the skills you will need to be a productive citizen and to succeed on the job, in college, or in the military. You will have to pass the HSPA to graduate from high school. If you do not pass the HSPA in March of your junior year, you will have the opportunity to take the HSPA again in October and March of your senior year. In addition, you will begin a Special Review Assessment for the HSPA in the fall of your senior year. The HSPA SRA is an alternative assessment that will enable you to show whether or not you have mastered the same knowledge and skills assessed by the HSPA.

The HSPA currently has two test sections, Language Arts Literacy and Mathematics. You will take the test over a three-day period for approximately two hours each day. Mathematics will be tested on Tuesday, March 1, and Language Arts Literacy will be tested on Wednesday and Thursday, March 2 and 3.

### **HOW WILL MY SCORES BE REPORTED?**

When you receive your HSPA scores, the report will show total scores in Language Arts Literacy and Mathematics. It will also show subtotal scores for the specific knowledge and skills measured in each content area. The total scores will be reported in one of three proficiency levels—Advanced Proficient, Proficient, or Partially Proficient. If you have not met the appropriate level of proficiency, your school may give you additional help to develop the skills you will need to pass the HSPA in the fall or spring of the twelfth grade.

### **WHAT WILL THE TEST BE LIKE?**

Like other tests you may have taken, the HSPA contains multiple-choice questions that require you to choose your response from among four answer choices (A, B, C, or D) and record it by darkening the appropriate choice in your answer folder. The test also contains open-ended questions that require you either to respond in your own words in written text, to draw a diagram, or to construct a numerical response.

Your responses to all multiple-choice and open-ended questions must be recorded in a separate answer folder. Information recorded in your test booklet or on scratch paper does not count toward your score.

## Language Arts Literacy

The Language Arts Literacy Section of the test includes reading and writing activities that will measure your achievements in interpreting, analyzing, and critiquing text. The reading materials will require you to read passages selected from published books, newspapers, and magazines, and to respond to related multiple-choice and open-ended questions.

In addition, you will write two extended responses. One will be based on a picture prompt, and the second will be based on a persuasive prompt.

## Mathematics

The Mathematics Section of the test will measure your ability to solve problems by applying mathematical concepts. The areas to be tested are: number and numerical operations; geometry and measurement; patterns and algebra; and data analysis, probability, statistics, and discrete mathematics.

Most mathematics questions are multiple choice, which have a weight of one point each for correct answer choices. The open-ended questions, requiring you to construct and explain your own written or graphic responses, receive a score from 0 to 3. For 3 points, a response must show complete understanding of a problem's concepts and have a clear, effective explanation. For 2 points, there must be a nearly complete understanding of a problem's concepts, but the response may have minor errors. A 1-point response shows limited understanding of a mathematical concept and has an incomplete explanation of how the problem was solved. A 0-point response shows insufficient understanding of the concept and may contain major errors.

You will be provided with a calculator when you take the test, and will receive a Mathematics Reference Sheet that contains formulas and other useful information you can use during the test.

### HOW CAN I PREPARE MYSELF TO TAKE THE HSPA?

- Relax. You will think more clearly if you are relaxed when you take a test. Dress comfortably on the day of the test so that you are at ease and not distracted. Put all your other activities on hold so that you can give all your attention and energy to taking the test and doing well.
- Get a good night's sleep the night before the test. Start your day off with a good breakfast so that you have plenty of energy to take the test.
- Don't cram. The skills measured by the HSPA are learned over a long period of time.
- Think positively. Believe that you will do your very best. Be confident of your ability.
- Read the directions carefully before beginning each part of the test. If you understand what you are supposed to do, it will save time and help you avoid careless mistakes.

- Read each question carefully. Try to answer the question before you look at the responses. If you find your answer there, mark that response. If not, ask yourself whether your answer is reasonable. Reread the question, keeping the responses in mind. Make sure that you understand what the question is asking.
- Make sure that your answers are reasonable. Do you understand what the question is asking? Have you made use of all the relevant information provided to answer the question correctly? Does your response answer the question? Did you choose the best answer among those listed?
- If you aren't sure of the answer to a question, try to eliminate some of the responses. Think about the reasons why you were able to eliminate some of the choices. These reasons may provide you with the information you need to choose the correct answer. If you can eliminate some of the choices, select the remaining answer choice that makes the most sense.
- Skip a question and go on to the next one if you have no idea of the answer. Spending too much time on one question might keep you from having enough time to answer others that you do know. You should not leave any question unanswered. If there is time, you should come back to it later at the end of that part of the test.
- Pace yourself during the test. Budget your time so that you have a chance to answer all of the questions. Your teacher will periodically let you know the time remaining in the part of the test you are taking.
- Fill in your answer folder carefully. Make sure that you record all your responses in your separate answer folder in the right spaces. No credit will be given for anything written in the test booklet. You may know the answer to a question, but if you do not mark your answer in the right place, you will not receive credit for your answer.
- Check your answers as you take the test. Make sure that you have chosen the response that best answers the question. Checking your answers as you work through the test will save time later in rethinking a question. Check your answer folder to make sure that you have darkened the correct answer space.
- Some questions require more planning than others. This is especially true of open-ended questions and writing tasks. First, outline the steps required to respond to the question. Then, identify related information and eliminate non-related information when you can.

## WHAT WILL THE HSPA LOOK LIKE?

The rest of this booklet will give you an idea of what the HSPA materials are like.

### Language Arts Literacy

New Jersey's Core Curriculum Content Standards identify five categories of Language Arts Literacy: speaking, listening, writing, reading, and viewing. These five activities are essential aspects of our everyday lives and critical to what we think, learn, communicate, and create.

The HSPA provides a variety of writing and reading activities and texts that will allow you to demonstrate your skills and knowledge in using language arts literacy. The assessment presents two types of reading passages, narrative and persuasive texts, that are followed by a set of 10 multiple-choice and two open-ended questions for each passage. You will be required to read two narrative passages and two persuasive passages. The HSPA also provides two different types of writing prompts. For the two writing tasks, you will be provided with blank pages for prewriting in either your test booklet (for the picture prompt) or in a separate writing task folder (for the persuasive prompt). Use this space to plan your ideas. Then use the lined pages in your answer folder for your written response. In addition, you will receive a copy of the Writer's Checklist/Revising-Editing Guide for each writing activity. The checklist and the guide list important points for you to remember as you write, critically read, and revise your writing. As you complete these different sections of the HSPA, you will be demonstrating your skills in using language for thinking, learning, and communicating.

The HSPA Language Arts Literacy activities are sequenced to give you varying experiences in using language for different kinds of tasks, just as you do in your everyday life. As preparation for taking the HSPA Language Arts Literacy assessment, read through the sample test materials in this booklet to familiarize yourself with the sequence and content of each test section. Also acquaint yourself with the type of scoring procedure and criteria that will be used to assess your demonstrated skills. This preview will help you understand what each task involves and how your work will be evaluated. Copies of the rubrics that will be used to score your writing are included on pages 14–16 of this booklet.

Following are sample materials illustrating the content of the HSPA Language Arts Literacy assessment. The reading passages are only excerpts from stories and articles that were chosen to illustrate the HSPA texts. Complete versions of these passages are printed in the *Directory of Test Specifications and Sample Items for the Elementary School Proficiency Assessment (ESPA), Grade Eight Proficiency Assessment (GEPA), and High School Proficiency Assessment (HSPA) in Language Arts Literacy*. That document is available in your school district.

This booklet includes an example of each writing activity and both types of questions for each reading passage. As you read through the following pages, notice that the scoring procedure for each open-ended item or activity is identified to help you understand how your work will be scored.

One writing task begins with a picture that serves as a prompt for a story. The setting and characters, when applicable, portrayed in the picture suggest an event or relationship that you are invited to develop and describe through a story. In this activity you may speculate about what has happened, what is happening, or what will happen at some other time. Use your imagination to create a good story that is detailed and vivid. The answer folder will provide two lined pages for your writing.



Robert Lewis/Robert Lewis Photography/New York, N.Y.

### WRITING TASK

An ancient proverb says, "A picture is worth a thousand words." Regardless of the artist's original intent, what we see in the picture can be very different from what others see. What story does this picture tell you? Use your imagination and experience to speculate about what is happening. Then write your story.

SCORING PROCEDURE: Registered Holistic Scoring Rubric - Page 11

A second writing activity will introduce a controversial issue that you will address in a persuasive letter or essay. This writing prompt is contained in a separate Persuasive Writing Task Folder rather than in the test booklet. Following is an example of a writing prompt that focuses on a controversial issue.

## **PERSUASIVE WRITING TASK**

### **Writing Situation**

In recent years, business representatives have expressed concern about the skills of students entering the workforce. Responding to these concerns, state legislators have enacted a law that establishes high educational standards for all students. Now the state legislature is considering enacting a law that would prohibit students from participating in any after-school activity after 6 pm. They believe this law would ensure that students have adequate time to study and complete daily homework assignments. However, many people believe this law would be unfair, and the proposed legislation has become a controversial issue in communities across the state.

Your social studies teacher has asked students to write an essay explaining their opinions of this controversial issue. What is your point of view? How would this legislation affect you and other students in your school?

### **Directions for Writing**

Write an essay either supporting or opposing the proposed legislation to prohibit students from participating in any after-school activity after 6 pm. Use facts, examples, and other evidence to support your point of view.

The answer folder will provide four lined pages for this writing task. In addition, you will have blank pages in the Persuasive Writing Task Folder to use for planning your writing. The planning space is for your benefit because it gives you a place to brainstorm and organize your ideas before you begin writing. Readers who score your writing will consider the organization and elaboration of key ideas and details as well as grammar and sentence structure. Only your writing in your answer folder will be scored.

**SCORING PROCEDURE: Registered Holistic Scoring Rubric - Page 11**

**NEW JERSEY  
HIGH SCHOOL  
PROFICIENCY ASSESSMENT**

**Writer's Checklist**

**Important Points to  
Remember as You  
Write and Critically Read  
to Revise/Edit Your Writing**

**CONTENT/ORGANIZATION**

- \_\_\_\_\_ 1. Focus on your purpose for writing and your audience.
- \_\_\_\_\_ 2. Develop a clear topic or central idea.
- \_\_\_\_\_ 3. Support your ideas with details, explanations, and examples.
- \_\_\_\_\_ 4. Put your ideas in the order that best communicates what you are trying to say.

**SENTENCE CONSTRUCTION**

- \_\_\_\_\_ 5. Use clear and varied sentences.

**USAGE**

- \_\_\_\_\_ 6. Use words correctly.
- \_\_\_\_\_ 7. Use varied and vivid vocabulary.

**MECHANICS**

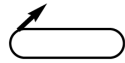
- \_\_\_\_\_ 8. Capitalize, spell, and punctuate correctly.
- \_\_\_\_\_ 9. Write neatly.

**NEW JERSEY STATE DEPARTMENT OF EDUCATION**



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## Revising/Editing Guide



shows where  
to move text

~~cross-out~~  
shows what  
to get rid of  
or change



shows what  
to insert



shows what  
text to add  
and where  
to add it

you may want to use editing

marks (when you revise and edit),  
insert change  
to move text, ^ text, or eliminate

text. Sometimes you may want to

add a sentence or paragraph. (A)

Whatever changes you make, be

sure to make your revisions and

editing marks clear to your readers.



If you want to add new text, label  
the new text with a letter or number.  
Then write the label to show where  
you are adding it.

What to consider when you revise and edit:

### CONTENT/ORGANIZATION

1. opening and closing
2. development of key ideas
3. logical progression of ideas
4. supporting details
5. transitions

### SENTENCE CONSTRUCTION

6. correct sentence structure (syntax)
7. varied sentence structure

### USAGE

8. correct verb tenses
9. subject/verb agreement
10. pronoun usage and agreement
11. word choice

### MECHANICS

12. spelling
13. capitalization
14. punctuation

The HSPA also presents a narrative reading passage, followed by 10 multiple-choice and two open-ended questions that relate to the text. The answer folder will provide a lined page for each open-ended response.

According to an old saying, “Good things are worth waiting for,” but how long should we wait? The following story explores this question through the experience of the two characters.

### The Telescope

by August Derleth

The door opened. The old man stood on the threshold looking out.

“It’s a boy,” he said to himself in mild surprise. “It’s Doc Grendon’s boy.”

“Grandpa said you had a telescope,” I said.

“A telescope,” he repeated wonderingly. He frowned a little, raising his head and looking squint-eyed out into the late winter afternoon. “Stopped snowing,” he said, nodding. “Well, now. . .” He paused, his jaws working as if he chewed away at something. He was tall and gaunt, with uncombed hair straggling out from underneath a kind of homemade stocking cap—an old stocking cut off and crudely sewed together again at the top. His nose was prominent and his mouth puckered. There was a thin beard on his chin. He wore overalls and a blue shirt that was open at the neck, so you could see his underwear. “I guess I did have a telescope once,” he said. “I remember I got it from Fred Hartley just before Fred sailed off. It came in a long box, directions and all, but I don’t know. . .” His sentence trailed away and died.

“Grandpa said you had it in that back room,” I said.

“Likely I have. He ought to know. D’you

them. I did not know what. Once I had got in, he seemed to regret letting me come. He stood there a little uncertainly, his pale blue eyes gazing at me, troubled, a frown on his forehead.

“I don’t know,” he said. “Maybe you could come back tomorrow, or next week sometime. I don’t know where it is.”

“We could look for it,” I said. I would not tell him that Grandfather Grendon had told me I must insist on seeing the telescope or else old man Corey would keep right on putting me off, day after day, and never take the trouble to look.

“Well,” he said, “well—it’s bound to be dark in there. No fire, either. Don’t have a light there. We’d have to use a lantern.”

“Where’s the lantern?” I asked.

He was baffled now. He gave a hitch or two to his overalls, took a deep breath, and let his shoulders sag. “Well, now,” he muttered. “Well, now. Let me see.”

The lantern was in plain sight on top of a shelf over the wood box, and he found it at last. He carried it over to the table and looked at it as if he hoped there might be something wrong with it. But it lit up right away. He picked it up, and behind the door, put on his

1. The purpose of memories in this story is to

- A. describe true love.
- 😊 B. reveal missed opportunities.
- C. tell an engaging story.
- D. find a telescope.

2. The experience with the telescope has a profound effect on the boy and Old Man Corey.

- What does each character see, and what is his response to what he sees in the telescope?
- How will the experience with the telescope change each of them?

Use information from the story to support your response.

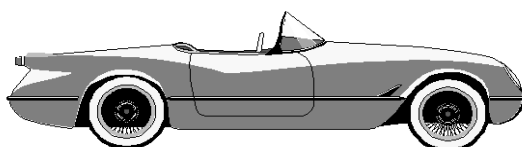
You will also read a persuasive passage and then answer 10 multiple-choice and two open-ended questions that relate to and extend your understanding of the text. The answer folder will provide a lined page for each open-ended response.

Nowadays, many teenagers believe they need to have after-school jobs that enable them to afford cars and other expenses, but in this article the writer presents a different perspective of what teenagers' primary job should be.

### Hold Your Horsepower

by Lyla Fox - *Newsweek*, March 25, 1996

Folks in the small Michigan town where I grew up, revere the work ethic. Our entire culture lauds those who are willing to work their tails off to get ahead. Though there's nothing wrong with hard work, I suggest that our youngsters may be starting too young and for all the wrong reasons.



Increasingly, I identify with Sisyphus<sup>1</sup> trying to move that stone. There are more mornings than I would like to admit when many of my students sit with eyes glazed or heads slumped on their desks as I try to nurture a threatening-to-become-extinct interest in school. These are not lazy kids. Many are high achieving 16- and 17-year olds who find it tough to reconcile 7:30 a.m. classes with a job that winds down at 10:30 p.m. or later.

"What's wrong?" I asked a student who once diligently completed his homework assignments. He groggily grunted an answer. "I'm tired. I didn't get home until 11 p.m." Since we end up working most of our adult life, my suggestion to the class was to forego the job and partake of school—both intra- and

our "no car in high school" dictate. When he needed to drive, we made sure he could borrow our car. Our Oldsmobile 88, however, didn't convey the instant high-school popularity of a sporty Nissan or Honda.

Our son's only job was to do as well as he could in school. The other work, we told him, would come later.

Today I see students working more than the legally permitted number of hours to pay for their cars. I also see once committed students becoming less dedicated to schoolwork. Their commitment is to their cars and the jobs that will help them make those monthly payments.

Once cars and jobs enter the picture, it is virtually impossible to get students focused on school. "My parents are letting me get a car," one of my brightest students enthused a few months ago. "They say all I have to do is get a job to make the payments: *All*. I winced, saying nothing because parents' views are sacrosanct for me. I bit my cheeks to keep from saying how wrong I thought they were and how worried I was for her schoolwork. Predictably, during the new month her grades and attitude took a

1. Which of the following would the author most likely support?

- ☺ A. expenditures for new texts rather than a driver's education course
- B. jobs for students as long as they don't require cars
- C. financial rewards for earning good grades
- D. fewer intra- and extracurricular activities

2. A student in your high school has taken a position that s/he is willing to work in order to own a car. Based on the article, what arguments could be made to justify such a position?

- Clearly state your point of view.
- Provide at least two supporting details that would explain your opposition to the author's point of view.

Use information from the article to support your response.

# NEW JERSEY REGISTERED HOLISTIC SCORING RUBRIC (for GEPA and HSPA only)

In scoring, consider the grid of written language	Inadequate Command	Limited Command	Partial Command	Adequate Command	Strong Command	Superior Command
Score	1	2	3	4	5	6
<b>Content and Organization</b>	<ul style="list-style-type: none"> <li>May lack opening and/or closing</li> </ul>	<ul style="list-style-type: none"> <li>May lack opening and/or closing</li> </ul>	<ul style="list-style-type: none"> <li>May lack opening and/or closing</li> </ul>	<ul style="list-style-type: none"> <li>Generally has opening and/or closing</li> </ul>	<ul style="list-style-type: none"> <li>Opening and closing</li> </ul>	<ul style="list-style-type: none"> <li>Opening and closing</li> </ul>
	<ul style="list-style-type: none"> <li>Minimal response to topic; uncertain focus</li> </ul>	<ul style="list-style-type: none"> <li>Attempts to focus</li> <li>May drift or shift focus</li> </ul>	<ul style="list-style-type: none"> <li>Usually has single focus</li> </ul>	<ul style="list-style-type: none"> <li>Single focus</li> </ul>	<ul style="list-style-type: none"> <li>Single focus</li> <li>Sense of unity and coherence</li> <li>Key ideas developed</li> </ul>	<ul style="list-style-type: none"> <li>Single, direct focus</li> <li>Unified and coherent</li> <li>Well-developed</li> </ul>
	<ul style="list-style-type: none"> <li>No planning evident; disorganized</li> </ul>	<ul style="list-style-type: none"> <li>Attempts organization</li> <li>Few, if any, transitions between ideas</li> </ul>	<ul style="list-style-type: none"> <li>Some lapses or flaws in organization</li> <li>May lack some transitions between ideas</li> </ul>	<ul style="list-style-type: none"> <li>Ideas loosely connected</li> <li>Transitions evident</li> </ul>	<ul style="list-style-type: none"> <li>Logical progression of ideas</li> <li>Moderately fluent</li> <li>Attempt compositional risks</li> </ul>	<ul style="list-style-type: none"> <li>Logical progression of ideas</li> <li>Fluent, cohesive</li> <li>Compositional risks successful</li> </ul>
	<ul style="list-style-type: none"> <li>Details random, inappropriate, or barely apparent</li> </ul>	<ul style="list-style-type: none"> <li>Details lack elaboration, i.e., highlight paper</li> </ul>	<ul style="list-style-type: none"> <li>Repetitious details</li> <li>Several unelaborated details</li> </ul>	<ul style="list-style-type: none"> <li>Uneven development of details</li> </ul>	<ul style="list-style-type: none"> <li>Details appropriate and varied</li> </ul>	<ul style="list-style-type: none"> <li>Details effective, vivid, explicit, and/or pertinent</li> </ul>
<b>Usage</b>	<ul style="list-style-type: none"> <li>No apparent control</li> <li>Severe/numerous errors</li> </ul>	<ul style="list-style-type: none"> <li>Numerous errors</li> </ul>	<ul style="list-style-type: none"> <li>Errors/patterns of errors may be evident</li> </ul>	<ul style="list-style-type: none"> <li>Some errors that do not interfere with meaning</li> </ul>	<ul style="list-style-type: none"> <li>Few errors</li> </ul>	<ul style="list-style-type: none"> <li>Very few, if any, errors</li> </ul>
<b>Sentence Construction</b>	<ul style="list-style-type: none"> <li>Assortment of incomplete and/or incorrect sentences</li> </ul>	<ul style="list-style-type: none"> <li>Excessive monotony/same structure</li> <li>Numerous errors</li> </ul>	<ul style="list-style-type: none"> <li>Little variety in syntax</li> <li>Some errors</li> </ul>	<ul style="list-style-type: none"> <li>Some variety</li> <li>Generally correct</li> </ul>	<ul style="list-style-type: none"> <li>Variety in syntax appropriate and effective</li> <li>Few errors</li> </ul>	<ul style="list-style-type: none"> <li>Precision and/or sophistication</li> <li>Very few, if any, errors</li> </ul>
<b>Mechanics</b>	<ul style="list-style-type: none"> <li>Errors so severe they detract from meaning</li> </ul>	<ul style="list-style-type: none"> <li>Numerous serious errors</li> </ul>	<ul style="list-style-type: none"> <li>Patterns of errors evident</li> </ul>	<ul style="list-style-type: none"> <li>No consistent pattern of errors</li> <li>Some errors that do not interfere with meaning</li> </ul>	<ul style="list-style-type: none"> <li>Few errors</li> </ul>	<ul style="list-style-type: none"> <li>Very few, if any, errors</li> </ul>

<b>NON- SCORABLE RESPONSES</b>	NR = No Response	Student wrote too little to allow a reliable judgment of his/her writing.
	OT = Off Topic/ Off Task	Student did not write on the assigned topic/task, or the student attempted to copy the prompt.
	NE = Not English	Student wrote in a language other than English.
	WF = Wrong Format	Student refused to write on the topic, or the writing task folder was blank.

Content/ Organization	Usage	Sentence Construction	Mechanics
<ul style="list-style-type: none"> <li>Communicates intended message to intended audience</li> <li>Relates to topic</li> <li>Opening and closing</li> <li>Focused</li> <li>Logical progression of ideas</li> <li>Transitions</li> <li>Appropriate details and information</li> </ul>	<ul style="list-style-type: none"> <li>Tense formation</li> <li>Subject-verb agreement</li> <li>Pronouns usage/agreement</li> <li>Word choice/meaning</li> <li>Proper Modifiers</li> </ul>	<ul style="list-style-type: none"> <li>Variety of type, structure, and length</li> <li>Correct construction</li> </ul>	<ul style="list-style-type: none"> <li>Spelling</li> <li>Capitalization</li> <li>Punctuation</li> </ul>

Note: All unscorable responses, (NSRs), with the exception of NR, must be coded by the Scoring Director.

## READING OPEN-ENDED SCORING RUBRIC

**Sample Task:** The author takes a strong position on voting rights for young people. Use information from the text to support your response to the following.

- \*Requirements:**
- Explain the author’s position on voting.
  - Explain how adopting such a position would affect young people like you.

Points	Criteria
<b>4</b>	A 4-point response clearly demonstrates understanding of the task, completes all requirements, and provides an insightful explanation/opinion that links to or extends aspects of the text.
<b>3</b>	A 3-point response demonstrates an understanding of the task, completes all requirements, and provides some explanation/opinion using situations or ideas from the text as support.
<b>2</b>	A 2-point response may address all of the requirements, but demonstrates a partial understanding of the task, and uses text incorrectly or with limited success resulting in an inconsistent or flawed explanation.
<b>1</b>	A 1-point response demonstrates minimal understanding of the task, does not complete the requirements, and provides only a vague reference to or no use of the text.
<b>0</b>	A 0-point response is irrelevant or off-topic.

**\*Requirements for these items will vary according to the task.**

# Mathematics

The Mathematics Section of the HSPA assessment is divided into four standards. Each standard reflects knowledge and skills specified in New Jersey's Core Curriculum Content Standards.

## High School Proficiency Assessment (HSPA) Mathematics Clusters

### 1. Number and Numerical Applications

- A. Number Sense
- B. Numerical Operations
- C. Estimation

### 2. Geometry and Measurement

- A. Geometric Properties
- B. Transforming Shapes
- C. Coordinate Geometry
- D. Units of Measurement
- E. Measuring Geometric Objects

### 3. Patterns and Algebra

- A. Patterns and Relationships
- B. Functions
- C. Modeling
- D. Procedures

### 4. Data Analysis, Probability, and Discrete Mathematics

- A. Data Analysis (Statistics)
- B. Probability
- C. Discrete Mathematics—Systematic Listing and Counting
- D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms

## Types of Questions

Many of the multiple-choice (MC) questions on the HSPA Mathematics test assess a level of cognitive processes that is higher than the cognitive processes assessed by the questions on a traditional multiple-choice test. It will take you an average of between one and two minutes to answer each MC question. The answers are computer scored and have a weight of one point each.

Open-ended (OE) questions require you to construct your own written or graphical responses and to explain your responses. It will take approximately ten minutes to answer each OE question. Your responses are hand scored on a scale from 0 to 3.

The general scoring guide on page 16 was created to help readers score open-ended questions consistently. This scoring guide is used by the trained readers who will score the Mathematics open-ended questions on the HSPA.

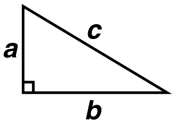
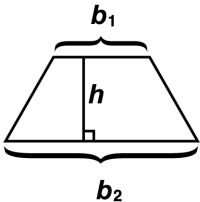
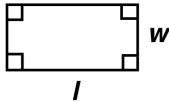
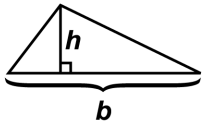
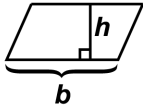
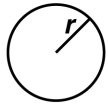
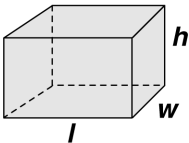
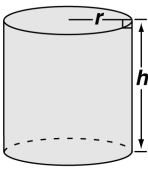
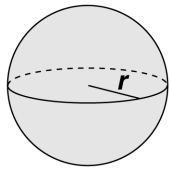
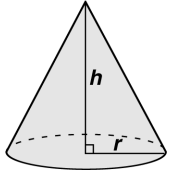
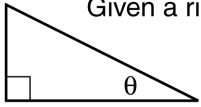
The following table shows you how many multiple-choice and open-ended questions to expect.

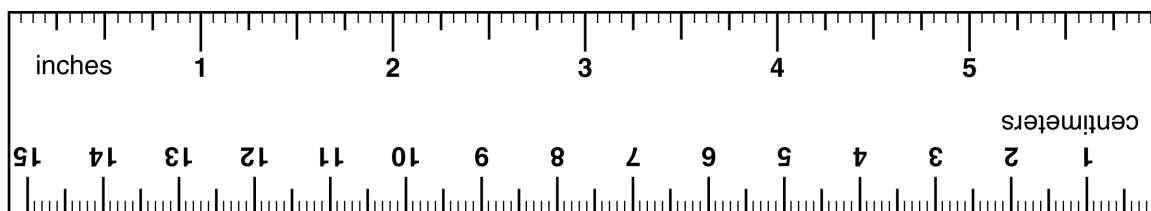
Question Type	Number of Questions
	11th Grade
MC	40
OE	8

You will be provided with a Mathematics Reference Sheet that contains a ruler, geometric shapes, formulas, and other information you may find useful as you take the test. You will also be provided with a calculator to help you solve problems.

# HIGH SCHOOL PROFICIENCY ASSESSMENT

## MATHEMATICS REFERENCE SHEET

<b>Pythagorean Formula</b> $c^2 = a^2 + b^2$ 	<b>Trapezoid</b> $\text{Area} = \frac{1}{2}h(b_1 + b_2)$ 	60 seconds = 1 minute 60 minutes = 1 hour 24 hours = 1 day 7 days = 1 week 52 weeks = 1 year
<b>Rectangle</b> $\text{Area} = lw$ $\text{Perimeter} = 2(l + w)$ 	<b>Triangle</b> $\text{Area} = \frac{1}{2}bh$ 	12 inches = 1 foot 3 feet = 1 yard 36 inches = 1 yard 5,280 feet = 1 mile 1,760 yards = 1 mile
<b>Parallelogram</b> $\text{Area} = bh$ 	<b>Circle</b> $\text{Area} = \pi r^2$ $\text{Circumference} = 2\pi r$ 	100 centimeters = 1 meter 1000 meters = 1 kilometer
<b>Rectangular Prism</b> $\text{Volume} = lwh$ $\text{Surface Area} = 2lw + 2wh + 2lh$ 	<b>Cylinder</b> $\text{Volume} = \pi r^2 h$ $\text{Surface Area} = 2\pi rh + 2\pi r^2$ 	8 fluid ounces = 1 cup 2 cups = 1 pint 2 pints = 1 quart 4 quarts = 1 gallon 1000 milliliters (mL) = 1 liter (L)
<b>Sphere</b> $\text{Volume} = \frac{4}{3}\pi r^3$ $\text{Surface Area} = 4\pi r^2$ 	<b>Cone</b> $\text{Volume} = \frac{1}{3}\pi r^2 h$ 	16 ounces = 1 pound 1000 milligrams = 1 gram 100 centigrams = 1 gram 10 grams = 1 dekagram 1000 grams = 1 kilogram
The sum of the measures of the interior angles of a triangle = $180^\circ$ The measure of a circle is $360^\circ$ or $2\pi$ radians		$\pi \approx 3.14$ or $\frac{22}{7}$
Given a right triangle:  $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$		Given the points $(x_1, y_1)$ , $(x_2, y_2)$ , <b>Distance between two points:</b> $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ <b>Slope Formula:</b> $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ <b>Slope-intercept form of a line:</b> $y = mx + b$ Distance = rate $\times$ time
<b>Interest</b> = principal $\times$ rate $\times$ time <b>Simple Interest Formula:</b> $A = p + prt$ <b>Compound Interest Formula:</b> $A = p\left(1 + \frac{r}{n}\right)^{nt}$ $A$ = amount after $t$ years; $p$ = principal; $r$ = annual interest rate; $t$ = number of years; $n$ = number of times compounded per year The number of <b>combinations</b> of $n$ elements taken $r$ at a time is given by $\frac{n!}{(n-r)!r!}$ The number of <b>permutations</b> of $n$ elements taken $r$ at a time is given by $\frac{n!}{(n-r)!}$		



## **Scoring Guide for Mathematics Open-Ended (OE) Questions (Generic Rubric)**

### **3-Point Response**

The response shows complete understanding of the problem's essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

### **2-Point Response**

The response shows nearly complete understanding of the problem's essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

### **1-Point Response**

The response shows limited understanding of the problem's essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

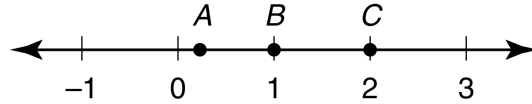
### **0-Point Response**

The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution, or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.

The generic rubric above is used as a guide to develop specific scoring guides or rubrics for each of the open-ended (OE) questions that appear on the New Jersey statewide assessments in Mathematics. The generic rubric helps ensure that students are scored in the same way for the same demonstration of knowledge and skills regardless of the test question.

## HSPA MATHEMATICS SAMPLE QUESTIONS

### Standard 1, Strand A



Where on the number line above would you locate the point that corresponds to the product of the coordinates corresponding to  $A$  and  $C$ ?

- A. to the right of  $C$
- \*B. between  $A$  and  $B$
- C. to the left of  $A$
- D. between  $B$  and  $C$

Rationale:

$A$  is a number that is less than  $\frac{1}{2}$  and greater than 0.  $A$  is multiplied by  $C$  (which is the whole number 2). This operation will produce a product that is less than 1 but greater than  $A$ .

## Standard 1, Strand A

Ray Hunter saved \$2,500 for a trip to the Grand Canyon. Ray estimates that he will have the following expenses on his trip:

Round-trip airfare	\$800.00
Transport to or from airport (one way)	\$ 22.00
Rental car (weekly)	\$137.00
Motel room (daily)	\$ 95.00
Meals (daily)	\$ 60.00
Extras (trail and helicopter rides, museums, gifts, etc.)	\$300.00

Ray's \$2,500 must cover all his expenses. What is the greatest number of days that Ray can plan to stay at the Grand Canyon? Show how you arrived at your answer.

Rationale:

Ray can stay for 7 days (1 week) at the Grand Canyon.

$\$800.00 + 2(\$22.00) + \$137.00 + 7(\$95.00) + 7(\$60.00) + \$300.00 = \$2,366.00$  for 7 days, leaving a balance of \$134.00.

## Standard 1, Strand B

The original ticket price of a shirt is \$25.99. During a clearance sale, the shirt sells for 40% off the original ticket price, with an additional 25% off the reduced price taken at the cash register.

- Rounded to the nearest cent, what is the price paid by the customer?
- What price would the customer have paid if this shirt were sold at a one-time reduction of 65% off the original price?
- Why didn't the store simply sell this shirt at 65% off the original sticker price?

Rationale:

Amount of 40% discount:  $\$25.99 * 0.40 = \$10.396 \approx \$10.40$

Price after 40% discount:  $\$25.99 - \$10.40 = \$15.59$

Amount of 25% discount:  $\$15.59 * 0.25 = \$3.897 \approx \$3.90$

Price after 25% discount:  $\$15.59 - \$3.90 = \$11.69$

(\$11.70 is acceptable if you round at the end instead of after each step.)

Amount of 65% discount:  $\$25.99 * 0.65 = \$16.894 \approx \$16.894 \approx \$16.89$

Price after 65% discount:  $\$25.99 - \$16.89 = \$9.10$

By breaking up the 65% discount into 40% and 25% discounts, the store was able to sell the shirt at a higher price than it would have if the store sold the shirt at 65% off the original price.

## Standard 1, Strand C

Jenna got an answer of about 2.88 when she entered 24 on her calculator and pressed the  $\sqrt[3]{\phantom{x}}$  key. As usual, she stopped to think briefly about whether or not her calculator's answer was reasonable. Which of the following statements is the most likely explanation for her to believe that her calculator's answer is or is not reasonable?

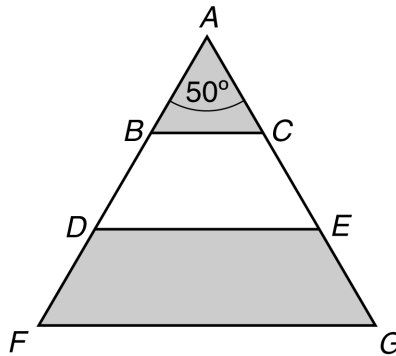
- A. It is not reasonable, because the answer should be a whole number.
- \*B. It is reasonable because 2 cubed is 8 while 3 cubed is 27.
- C. It is not reasonable because the answer should be only slightly more than 2.
- D. It is reasonable, because 24 is an even number.

Rationale:

The correct answer is B. The  $\sqrt[3]{8} = 2$  and the  $\sqrt[3]{27} = 3$ . Therefore, when calculating the  $\sqrt[3]{24}$ , it is reasonable to expect the answer to lie between 2 and 3, and to be closer to 3 than to 2 because 24 is closer to 27 than it is to 8.

## Standard 2, Strand A

For a sewing project, Tanya cut isosceles triangles from a striped piece of material where the stripes are parallel. The vertex angle of the isosceles triangle is  $50^\circ$  and  $\overline{BC}$  is parallel to the base.



Find the measure of  $\angle BCE$  as shown in the diagram.

- A.  $50^\circ$
- B.  $65^\circ$
- \*C.  $115^\circ$
- D.  $130^\circ$

Rationale:

Since the triangle is isosceles and the vertex angle is given to be  $50^\circ$ , the two remaining angles must be  $65^\circ$ .  $180^\circ = 50^\circ + x + x$ ;  $x = 65^\circ$

Since  $\angle ACE$  is a straight line, its measure is  $180^\circ$ . Therefore,  $m\angle ACB + m\angle BCE = 180^\circ$ .

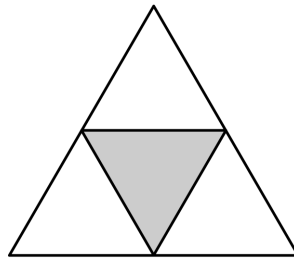
Since  $\overline{BC}$  and  $\overline{FG}$  are parallel and  $\overline{AG}$  intersects both  $\overline{BC}$  and  $\overline{FG}$ ,  $m\angle ACB = m\angle AGF = 65^\circ$ .

To solve for  $m\angle BCE$ , use the following:  $180^\circ - m\angle ACB = 180^\circ - 65^\circ = 115^\circ$ .

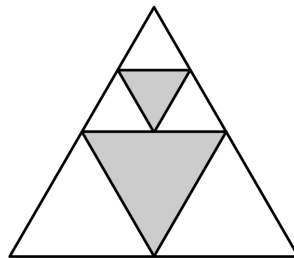
## Standard 2, Strand B

A design follows this pattern: an equilateral triangle is divided into 4 congruent triangles as shown below in Stage 1. Then the top triangle is divided into 4 congruent triangles, and the pattern repeats for each stage. In Stage 2, what is the ratio of the area of the larger shaded triangle to the area of the smaller shaded triangle?

Stage 1



Stage 2

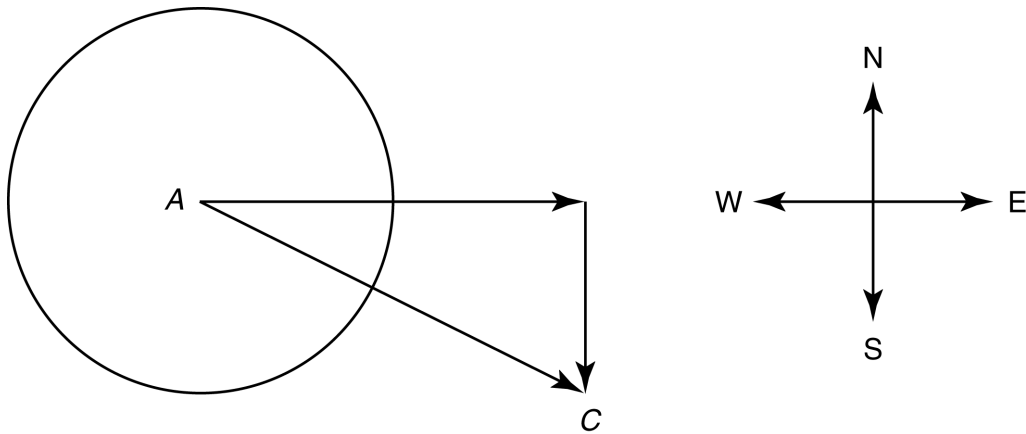


- \*A. 4:1
- B. 3:1
- C. 2:1
- D. 1:4

Rationale:

Since the initial equilateral triangle was divided into 4 congruent triangles, the shaded region in Stage 1 has an area equal to each of the unshaded triangles. In Stage 2, the top unshaded triangle was divided into 4 congruent triangles. Therefore, the area of the smaller shaded triangle is  $\frac{1}{4}$  the area of the top unshaded triangle. Since the top unshaded triangle is equal in size to the shaded triangle in Stage 1, the area of the smaller shaded triangle to the area of the larger shaded triangle is 1:4. Therefore, the area of the larger shaded triangle to the area of the smaller shaded triangle is 4:1.

**Standard 2, Strand C**



Michelle was fishing in her canoe at point  $A$  in the lake depicted above. After trying to fish there, she decided to paddle due east at a steady speed of 10 miles per hour. As she paddled, a wind blowing due south at 5 miles per hour caused a change in her direction. To the nearest tenth of a mile, what is the velocity, represented by vector  $AC$ , of her canoe?

- A. 8.6 miles per hour
- B. 10 miles per hour
- \*C. 11.2 miles per hour
- D. 17.2 miles per hour

Rationale:

$$10^2 + 5^2 = (AC)^2$$

$$100 + 25 = (AC)^2$$

$$125 = (AC)^2$$

$$\sqrt{125} = AC$$

$$11.18 \approx AC, \text{ rounded to the nearest tenth} = 11.2$$

## Standard 2, Strand C

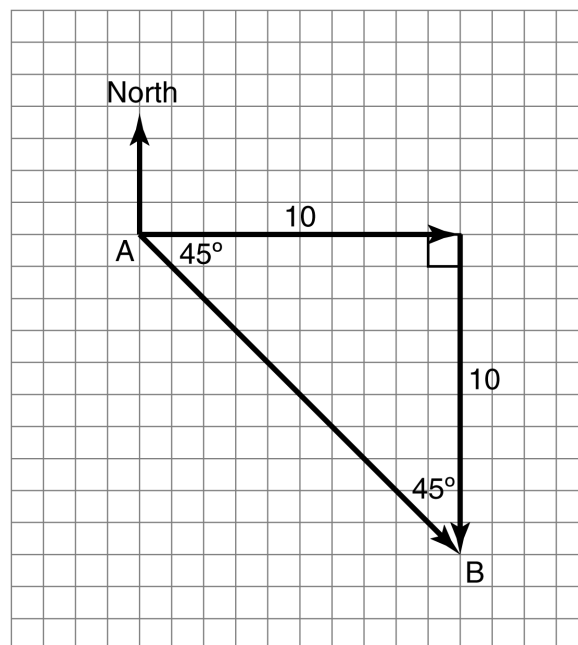
A car starts at point A, travels 10 miles east, and then turns and travels 10 miles south to reach point B.

- Using the grid provided in your answer folder, make a scale drawing using vectors to show the movement of the car, starting from point A.
- Draw a vector that would show the direct path from point A to point B. What would be the approximate number of miles the car could have traveled along this direct path?
- Approximately how many degrees from north would this direct path be? Show your work or explain how you arrived at your answer.

Rationale:

The drawing forms a right isosceles triangle that is 10 units on each side. The measure of the direct path from A to B is  $\sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2} \approx 14.14$  miles.

Since it is a right isosceles triangle, the interior angles are 45-45-90. Since north is  $90^\circ$  at point A and the directed path is  $45^\circ$  from the first 10-mile path A travels, then the directed path is  $90^\circ + 45^\circ = 135^\circ$  from north.



## Standard 2, Strand D

A chemistry measurement shows .02760 grams of sodium chloride in a beaker. How many significant digits are in this measurement?

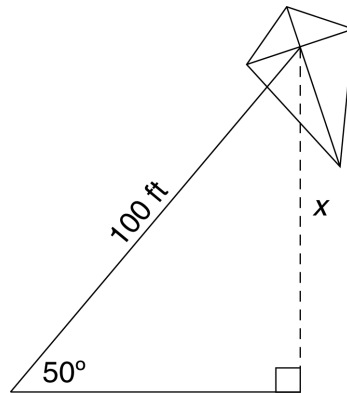
- A. 5
- \*B. 4
- C. 3
- D. 2

Rationale:

Significant digits are those numbers that make a contribution to a value. The numbers that are significant in .02760 are 2, 7, 6, and 0. The zero immediately following the decimal point (known as the leading zero) is not significant. Any non-zero digit is significant (2, 7, and 6). The zero at the end of the number (known as the trailing zero) is significant.

**Standard 2, Strand E**

Tawana is flying her kite, which is at the end of a 100-ft string. The angle the string makes with the ground is 50 degrees.



Which equation below can be used to find the height,  $x$ , of the kite above the ground?

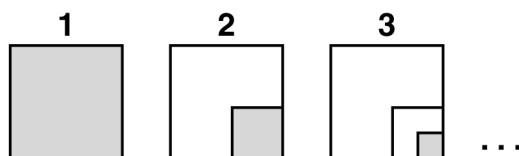
- A.  $\cos 50^\circ = \frac{x}{100}$
- \*B.  $\sin 50^\circ = \frac{x}{100}$
- C.  $\sin 50^\circ = \frac{100}{x}$
- D.  $\tan 50^\circ = \frac{x}{100}$

Rationale:

Since this forms a right triangle, side  $x$  is opposite the angle of 50 degrees and 100 ft is the hypotenuse of the right triangle. The sine of an angle is equal to the ratio of the opposite side of a right triangle to its hypotenuse.  $\sin 50^\circ = \frac{x}{100}$

### Standard 3, Strand A

A sequence of shaded squares is displayed below. One vertex of each shaded square, after the first, is at the center of a square the same size as the preceding shaded square.



The ratio of the area of the 10<sup>th</sup> shaded square to the area of the 12<sup>th</sup> shaded square is

- A.  $\frac{1}{16}$
- B.  $\frac{1}{4}$
- C.  $\frac{4}{1}$
- \*D.  $\frac{16}{1}$

Rationale:

The ratio of the area of any selected shaded square to the area of the next shaded square is  $\frac{4}{1}$ , that is, the area of the selected shaded square is 4 times the area of the next shaded square. Therefore, the ratio of the area of the 10<sup>th</sup> shaded square to the area of the 11<sup>th</sup> shaded square is  $\frac{4}{1}$ . The ratio of the 11<sup>th</sup> to the 12<sup>th</sup> is  $\frac{4}{1}$ . Thus, the ratio of the 10<sup>th</sup> to the 12<sup>th</sup> is  $\frac{4}{1} \cdot \frac{4}{1} = \frac{16}{1}$ .

### Standard 3, Strand A

Imagine that the table below continues, row after row, following the same pattern forever.

	Column A	Column B	Column C
Row 1	1	2	3
Row 2	4	5	6
Row 3	7	8	9
Row 4	10	11	12
Row 5	13	14	15
Row 6	16	?	?
Row 7	?	?	?
...	...	...	...
Row 100	?	?	?

- Complete the 6<sup>th</sup> and 7<sup>th</sup> rows.
- What numbers are in the 100<sup>th</sup> row?
- Write expressions for the numbers in the  $n^{\text{th}}$  row.
- In which row will the number 32 be found? Explain your answer.
- In which column will the number 32 be found? Explain your answer.
- In which column will the number 1,783 be found? Explain your answer.

Rationale:

Column C is always 3 times the row number (that is,  $3n$ )

Column B is always 3 times the row number less 1 (that is,  $3n - 1$ )

Column A is always 3 times the row number less 2 (that is,  $3n - 2$ )

Therefore, Row 6: 16, 17, 18  
Row 7: 19, 20, 21  
Row 100: 298, 299, 300

Since the number in Column C is always divisible by 3, and 32 is 1 less than 33 (which is divisible by 3), the number 32 appears in Column B. To find the row number, substitute the number 32 into the equation for Column B.

$$3n - 1 = 32$$

$$3n = 33 \text{ and } n = 11 \text{ (the 11}^{\text{th}} \text{ row)}$$

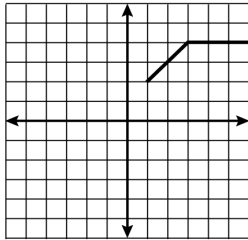
To find the column for 1,783, divide by 3. Note that the answer is 594 with a remainder of 1. The remainder indicates that the number occurs in the first column (Column A) of the next row, which is row 595. You can confirm your answer by using the formula for Column A.

$$3n - 2 = 1,783$$

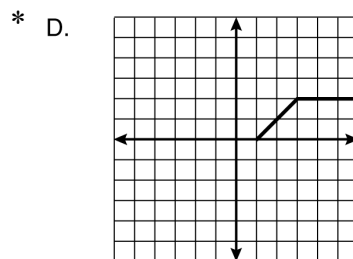
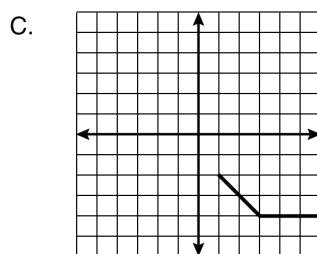
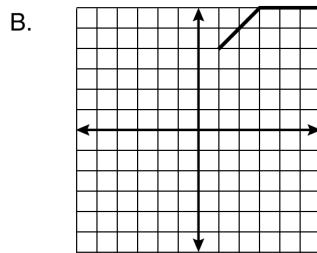
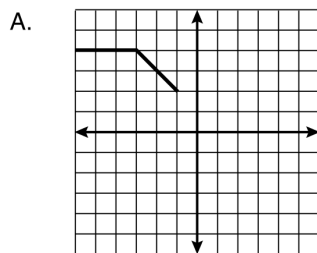
$$3n = 1,785 \text{ and } n = 595$$

### Standard 3, Strand B

The graph of a function,  $f(x)$ , is given below.



Which graph would represent  $f(x) - 2$ ?



Rationale:

The graph of the function  $f(x)$  is shown. So,  $f(x) - 2$  is the graph of  $f(x)$  translated 2 units in the negative  $y$  direction.

### Standard 3, Strand C

The basketball team scored 75 points in the final game of the season. During that time, the team made twice as many field goals as they did free throws. Each field goal is worth two points, and each free throw is worth one point. How many points did the basketball team make on free throws during the game?

Which of the following equations can be used to solve the problem given above?

A.  $2x + x = 75$

\*B.  $2(2x) + x = 75$

C.  $2x^2 = 75$

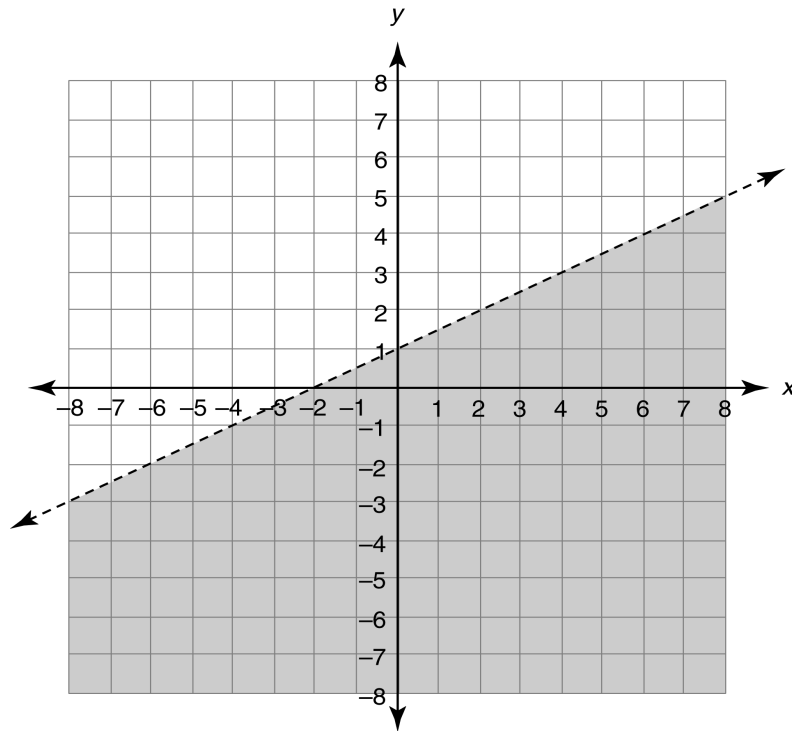
D.  $4x = 75$

Rationale:

$75 = 2x + x$  accounts for only the fact that the team made twice as many field goals as they did free throws. With the additional stipulation that each field goal is worth two points while each free throw is worth only one point, the equation  $2(2x) + x = 75$  should be used to find the number of points obtained by free throws.

**Standard 3, Strand D**

The graph below represents which of the following inequalities?



- A.  $y > \frac{1}{2}x + 1$
- \*B.  $y < \frac{1}{2}x + 1$
- C.  $x > \frac{1}{2}y + 1$
- D.  $x < \frac{1}{2}y + 1$

Rationale:

Since the line is a dashed line, its inequality is either  $y < \frac{1}{2}x + 1$  or  $y > \frac{1}{2}x + 1$ . Since the region shaded is below the dashed line, the inequality  $y < \frac{1}{2}x + 1$  is the correct answer.

## Standard 4, Strand A

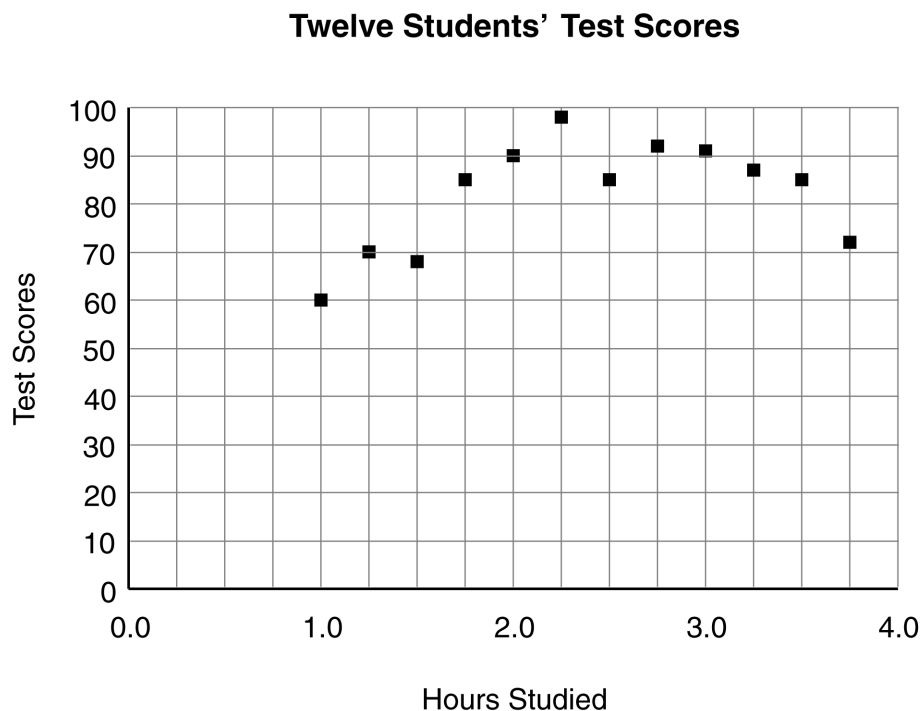
The data provided show test scores for twelve students and the number of hours they studied for the test during the three days prior to taking it.

Hours Studied	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0	3.25	3.5	3.75
Test Score	60	70	68	85	90	98	85	92	91	87	85	72

- On the grid provided in your answer folder, construct a scatter plot of this data.
- Does there appear to be a relationship between a student's test score and the time spent studying? Use the scatter plot to support your answer.
- Do any of the points appear to be outliers? Explain.

Rationale:

Student must draw a correct scatter plot of the data.



It looks as though a student's test score improves with more time spent studying up to a certain point (2.25 hours). After that time, a student's test score seems to diminish with more time spent studying. Answers as to which points are outliers may vary, as long as the response shows a clear understanding of the definition of outliers and supports the answers.

**Standard 4, Strand B**

Stacy has 6 marbles in a bag: a red, an orange, a yellow, a blue, a green, and a white. She randomly picks 2 marbles out of the bag one at a time without replacement. What is the probability that she will first pick the orange marble and then pick the blue marble?

- A.  $\frac{2}{6}$
- B.  $\frac{1}{6}$
- \*C.  $\frac{1}{30}$
- D.  $\frac{1}{36}$

Rationale:

The probability of picking the orange marble first is  $\frac{1}{6}$ . Since the orange marble is not being replaced, the probability of then picking the blue marble is  $\frac{1}{5}$ . Therefore, the probability of picking the orange marble and then the blue marble is  $\left(\frac{1}{6} \bullet \frac{1}{5}\right) = \frac{1}{30}$ .

#### Standard 4, Strand C

You and a group of 9 friends are playing pickup basketball in a local park. At the end of the game, if each player shakes hands with every other player once and only once, how many handshakes will there be?

- A. 36
- \*B. 45
- C. 81
- D. 100

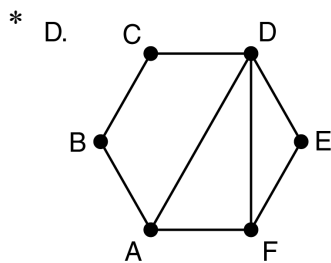
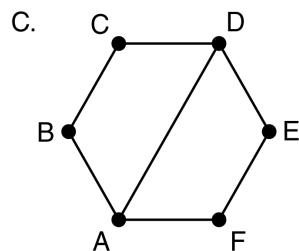
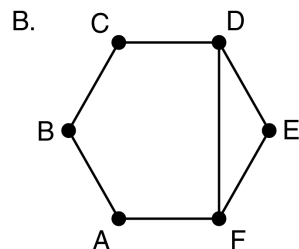
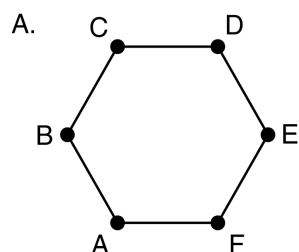
Rationale:

$\frac{10!}{(10-2)!2!} = \frac{90}{2} = 45$  OR  $\frac{10(10-1)}{2} = \frac{90}{2} = 45$  OR There are 10 players in all. The first player shakes hands with 9 other players. The second player shakes hands with 8 other players, having shaken hands with the first. The third player has to shake hands with only 7 other players, and so on.  $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ .

## Standard 4, Strand D

The following matrix represents an airline's direct travel routes between 6 cities – A, B, C, D, E, and F. Which diagram below can represent this direct-route matrix? (A 0 indicates no direct route between the two cities, or no travel is needed; a 1 indicates a direct route.)

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$



Rationale:

The columns should be labeled as A, B, C, D, E, and F, respectively, and the rows should be labeled similarly. The following routes exist: A to B, A to D, A to F, B to A, B to C, C to B, C to D, D to A, D to C, D to E, D to F, E to D, E to F, F to A, F to D, and F to E.

